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16. Abstract

The calculation results presented in this paper indicate that resonant interaction in triads in the presence of a pressure gradient is qualitatively similar to the case of nongradient flow. The amplitude of the plane wave is the primary factor determining mode discrimination and the evolution of the triplet. If the amplitude of the plane wave is greater than some sufficiently low threshold level, a broad spectrum of three-dimensional subharmonic perturbations will be excited in all the flows considered. In slower flows the amplitudes of the primary wave and subharmonic are equalized in a narrower range of Reynolds numbers and the rates of increase of both components are much higher. The above makes it possible to infer that a positive pressure gradient has a destabilizing effect on resonant interaction in triads, which makes the role of gradients decisive in subharmonic transitions on actual streamline bodies.

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I. Maslennikova

RESONANT EXCITATION OF SPATIAL PERTUBATIONS IN A BOUNDARY LAYER IN THE PRESENCE OF A PRESSURE GRADIENT

1. The process of a laminar-to-turbulent transition and the mechanisms which lead to boundary layer turbulence are not only fundamental problems in fluid mechanics but also urgent problems in engineering applications. An understanding and quantitative analysis of transition mechanisms would make it possible to reliably predict and control it.

Experiments conducted in the last 25 years have made it possible to understand that a transition to turbulence is not unambiguous and different variations of this transition are possible, but the occurrence of three dimensional phenomena in otherwise two dimensional flows constitutes an inherent feature of a transition. Two types of transitions were detected in a boundary layer on a plate, namely the so-called Klebanoff transition [1] and the subharmonic transition [2-5]. These types of transitions can be distinguished by their driving mechanisms. A K-transition occurs with the development of a very nonlinear two dimensional wave of sufficiently high intensity. A subharmonic transition occurs with perturbations of much lower intensity and is characterized by the excitation of a broad spectrum of low frequency pertubations in the vicinity of a subharmonic of the fundamental frequency, the formation of certain spatial structures, and then the rapid rise of intensities of all perturbations.

Theoretical studies [6-9] revealed the existence of a resonance mechanism which can explain the powerful excitation of Tollmin-Schlichting waves. The model developed includes the interaction of a triplet of waves (a Tollmin-Schlichting plane wave and a pair of oblique subharmonics) and the collective interaction of plane and quasiplane waves with a packet of subharmonic waves [10-13]. It was demonstrated that a subharmonic transition is a result of the uncontrolled growth of three dimensional background perturbations and their subsequent effect on the fundamental frequency. This model has made it possible to explain all the characteristics of a resonance transition and obtain good quantitative agreement with experiment results.

All the aforementioned experimental and theoretical studies apply to a boundary layer on a flat plate. It is obvious that these data are completely inadequate for studying a transition on actual streamline bodies. Data on gradient flows are limited to calculations on the basis of linear theory [14]. In light of the fact that under actual conditions a subharmonic transition is most probable when the oncoming flow is not very turbulent, the task of studying the characteristics of a resonant mechanism of interaction in triads in gradient flows seems urgent. This is the purpose of this study. In it we use Faulkner-Skan profiles as model profiles.

2. The velocity field of perturbed flow in a boundary layer may be represented as

$$\vec{u} = (U + \varepsilon u',\ \varepsilon v',\ \varepsilon w'), \qquad (1)$$

where $\varepsilon(u', v', w')$ are respectively the {x, y, z} components of perturbation of an intensity $\varepsilon \ll 1$; {U(x, y), 0, 0} corresponds to primary flow with an accuracy to within terms $\sim 1/\mathrm{Re}$; $U = U_e(x)$ when $y \to \infty$. With an exponential change in velocity outside the boundary layer $U_e(x) = U_0 x^m$ the velocity profile U(y) inside the boundary layer can be determined from a Faulkner-Skan equation ($U = \Phi'_\eta$):

$$\Phi_{\eta\eta\eta} + \frac{m+1}{2}\Phi\Phi_{\eta\eta} = m(\Phi_\eta^2 - 1) \qquad (2)$$

with boundary conditions

$$\Phi(0) = \Phi'(0) = 0, \quad \Phi \to 1 \text{ при } \eta \to \infty.$$

Here

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{v_e(x)}}}$$

is a nondimensional lateral coordinate. A natural nondimensional variable in the x-direction is

$$\bar{x} = x \Big/ \frac{\nu x}{v_e(x)} = \sqrt{\mathrm{Re}_x} \equiv \mathrm{Re}.$$

Within the framework of these conceptions, the perturbation of longitudinal velocity may be represented as follows, under the assumption of local parallelism of flow:

$$u'(x, y, z, t) = \sum_{j=1}^{3} A_j(\mathrm{Re})\, e^{\gamma_j t} u_j(y, \mathrm{Re}, k_j)\, e^{i\theta_j},$$
$$\theta_j = \int \alpha_j dx + k_j z - \omega_j t, \quad j = 1, 2, 3. \qquad (3)$$

$$\alpha_i, u_j, \omega_j + i\gamma_j = \Omega_j(\alpha_i, k_i, \mathrm{Re})$$

can be determined by means of an Orr-Sommerfeld eigenvalue problem.

A synchronized triad of the type described can be defined by the conditions

$$(\omega, \alpha, k, \gamma)_2 = (\omega, \alpha, -k, \gamma)_3, \quad \omega_{2,3} = 1/2\omega_1, \quad k_1 = 0.$$

For slowly changing complex amplitudes $A_j = a_j e^{i\psi_j}$

the following system of equations $\left(\frac{\partial}{\partial t} \equiv 0\right)$ is valid: $\left(v_1 \frac{\partial}{\partial x} - \gamma_1\right) A_1 = \varepsilon S_1 A_2 A_3 h_-,$

$$\left(v_{2,3} \frac{\partial}{\partial x} - \gamma_{2,3}\right) A_{2,3} = \varepsilon S A_1 A^*_{3,2} h_+,$$
$$a_j \psi_i \,|\, x = x_0 = a_{r_0}, \ \psi_j$$

In this case v_j are complex group velocities; the coefficients S_1 and S are expressed by the solutions of eigenvalue problems; the terms

$h \pm (\Delta\alpha)$ take into account low intensity interactions with an increase in desynchronization in the triad

$$(\Delta\alpha = \alpha_1 - \alpha_2 - \alpha_3): h_\pm = \frac{1}{X}\int_0^X e^{i\Delta}dx,$$

where

$$\frac{d}{dx}\Delta = \Delta\alpha,$$

and the averaging interval

$$X \gg \varepsilon^{-1}\Delta\alpha/\alpha.$$

When desynchronization is slight

$$\Delta\alpha/\alpha \sim \varepsilon$$

the factor under the integral may be removed, and

$$h \pm \approx e^i \int_{x_0}^{x} \Delta\alpha dx,$$

which agrees with the previously used formula [10-13]. If desynchronization is significant ($\Delta\alpha/\alpha \sim 1$), then $h \pm \to 0$. In this study we used the approximate formula

$$h_\pm \approx \frac{e^{i\Delta\alpha X} - 1}{i\Delta\widetilde{\alpha} X},$$

where

$$\widetilde{\Delta\alpha} = \frac{1}{x - x_0}\int_{x_0}^{x} \Delta\alpha dx.$$

System (4) is used to model the development of an isolated two dimensional (j = 1) Tollmin-Schlichting wave in the presence of a background of spatial subharmonics (j = 2, 3) for different gradient parameters $\beta = 2m/(m + 1)$. With the normalization used

$$\max_{0<y<\infty} |u_j(\mathrm{Re}, y, k_j)| = 1$$

the modulus of amplitude a_j is equal to the maximum longitudinal velocity of the jth component with respect to the thickness of the boundary layer.

As the wave evolves, the dimensional magnitudes of frequency and the z-component of the wave vector are maintained. Under this condition in the case of $\beta \neq 0$, the nondimensional frequency parameter $F = \omega\nu/v_e^2(x)$ and the nondimensional wave number k change with a charge in the Reynolds number:

$$F(\mathrm{Re}) = F(\mathrm{Re}_0)\left(\frac{\mathrm{Re}}{\mathrm{Re}_0}\right)^{-\beta}, \qquad k(\mathrm{Re}) = k(\mathrm{Re}_0)\left(\frac{\mathrm{Re}}{\mathrm{Re}_0}\right)^{1-\beta},$$

and Re_0 corresponds to a starting point of x_0 .

Below we give the results of calculations for two fixed values of frequency ω corresponding to two values of the frequency parameter:

$$F_1(\mathrm{Re}_0) = 1{,}15 \cdot 10^{-4} \text{ (I) и } F_1(\mathrm{Re}_0) = 0{,}315 \cdot 10^{-4} \text{ (II)}.$$

In both cases we examined Reynolds numbers whose origins (Re_o) lie in the vicinity of the maximum rates of increase for a given frequency. Both ranges are depicted schematically in Figure 1.

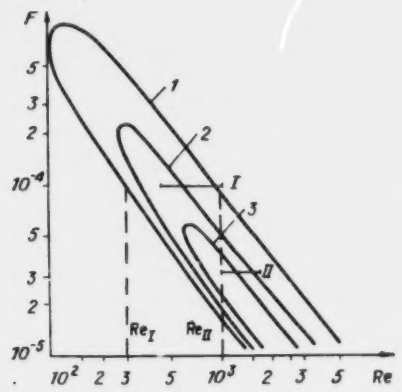


Figure 1. Ranges I and II of the Reynolds numbers in question

β: —0,1 (1), 0 (2), 0,1 (3).

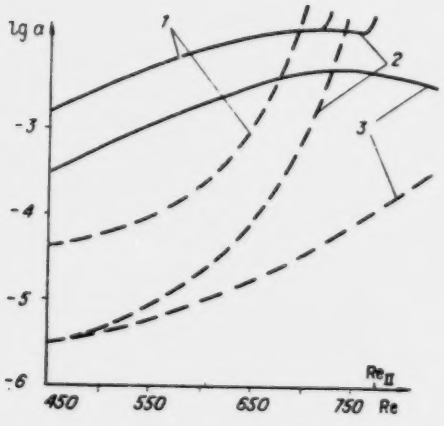


Figure 2. Curves of the increase of the components of the triplet

β=—0,1; Re_o=450; $F_1(Re_o) = 1{,}15 \cdot 10^{-4}$, b=0,275; X=500.

3. A number of qualitative properties of the evolution of coupled perturbations in a triad are independent of the gradient parameter and are determined by the resonant nature of interaction. Let us examine them using an example of a triplet in gradient flow with β = -0.1.

Typical curves of the increase of a T-S plane wave (solid lines) and an inclined subharmonic wave (dashed lines) are given in Figure 2. As in the case of nongradient flow (10), there exists a rather extended region of parametric pumping of the subharmonic of the plane wave, and the latter behaves linearly. After intensities have been equalized

$$(a = a_{2,3} \geq a_1)$$

where occurs a stage of nonlinear interaction leading to an abrupt increase in the intensities of all the components of the triplet (curves 1, 2). This is the property of nonlinear resonant instability which is "explosive" in nature and is important for application to the transition process. However this behavior of perturbations is manifested when initial amplitudes and interval lengths (Re_O , Re_{II}) are sufficient for equalizing intensities in the region of instability of the primary wave or somewhat to the right of it. In the opposite case the perturbations will be damped.

A decrease in initial amplitudes a_1 (Re_O) and a(Re_O) and the presence of a relative perturbation phase shift at the starting point
$\Delta\psi \neq 0$ ($\Delta\psi = \psi_1 - \psi_2 - \psi_3$) cause the "explosion" to shift downstream (curves 2, 3). The amplitude of the primary wave is the main factor determining the evolution of a triplet with a wide variation of other parameters.

4. In principle, the question of the excitation of three dimensionality in the type of transition in question seems quite clear. The excitation mechanism is parametric pumping of random background perturbations of a plane wave, but the structure of this three dimensionality requires its own explanation.

Experiments in a nongradient boundary layer [3-5] have indicated that three dimensionality in the pretransition region is manifested in the formation of certain spatial structures which can be detected by an imaging technique. They are characterized by a wavelength λ_x which is approximately twice that of the length of the plane wave and a certain ratio of wavelengths in longitudinal and lateral directions $\lambda_x/\lambda_z = k/\alpha_i$

On a plate, values of λ_x / λ_z ranging from 1.36 to 2.96 were obtained with different perturbation intensities and frequency parameters. In other words, from the background one can distinguish a predominant perturbation with a certain k which varies widely in relation to parameters.

In a number of studies [3, 5, 15] it was assumed that the resonant mechanism in symmetric triads is incapable of explaining this discrimination of a predominant mode. It is based on the idea that interaction is realized primarily in resonant triads ($\Delta\alpha = 0$). On a plate in resonant triads the relation $\lambda_x / \lambda_z \approx 1$ is fulfilled, which corresponds to perturbations propagated at an angle of approximately 45 degrees to the direction of flow, which is in contradiction with experiments. However the requirement for exact resonance, which does

not take into consideration the relationships of the coefficients of equation (4) to k is not necessary. The authors of [10] and [16] demonstrated that on a plate is excited an entire k spectrum of three dimensional perturbations whose peak corresponds to propagation angles of 50 to 63 degrees (or λ_x/λ_z = 1.4 to 2), which provides a complete explanation for experiment data. It seems natural that the same discrimination mechanism operates in gradient flows. Let us examine it in detail for two values of the gradient parameter: β = -0.1 and 0.1.

Figure 3 illustrates the relationships of desynchronizations $\Delta\alpha$ to Re for different adjusted wave number values

$$b\left(b = 10^3 \cdot \frac{k}{\mathrm{Re}^{1-\beta}}\right)$$

for β = 0 (solid curves) and β = -0.1 (dashed curves). Different values correspond to different points Re_{pes} where $\Delta\alpha$ = 0 is fulfilled. In the case of a zero pressure gradient this condition is fulfilled when

$$\xi = \mathrm{arctg}\,(k/\alpha_2) = 45 \div 48^\circ$$

and a decrease in b merely causes the point Re_{pes} to shift in the direction of higher Re. In contrast to this case, when β = -0.1 there exists a minimum value of b_{min} such that for smaller b the condition

$\Delta\alpha$ = 0 is nowhere fulfilled. The resonance points belonging to the (Re_{I} , Re_{II}) range correspond to angles ξ = 53 to 60 degrees. The adjusted numerical values are independent of frequency.

When β = 0.1 resonance conditions and the relationship of $\Delta\alpha$ to b are the same as in the case of nongradient flow. The local rate of increase of the subharmonic in the parametric region is a function of Re, b, and the amplitude of the plane wave:

$$\frac{1}{a}\frac{da}{dx} \equiv \sigma = \mathrm{Real}\,\{(\gamma_2 + \varepsilon S A_1 h_+)/v_2\}.$$

Figure 4 (solid curves 1-6) illustrates the relationship σ(b) for β = -0.1 at the point Re = 574 inside a curve of neutral stability (c.n.s.). Curve 1 corresponds to linear instability ($a_1 \equiv$ 0). At low primary wave amplitudes a_1 (Re_o) $\lesssim$ 0.015%, as in the linear case, planar subharmonics are the most unstable (curve 2). When a_1 (Re_o) $\approx$ 0.015% (curve 3), there appears a pronounced peak of the rate of increase when b $\neq$ 0 which corresponds to inclined subharmonics with k/α_2 = 1.37

(ξ = 53 degrees), which in turn correspond to resonance conditions in the triad. With an increase in a_1 (Re_o) in the range 0.015 to 0.4% (curves 3-5) the peak of σ (b) shifts toward $k/\alpha_2 \approx 1.6$, and with a further increase in a_1 (Re_o) its position remains unchanged (curves 5, 6). At point Re_{II} with the same initial amplitudes the ratio k/α_2 varies from 1.9 to 2.7.

Due to this relationship between predominant propagation angles and Re, during a process of parametric evolution three dimensional subharmonics with different, Re-dependent ratios k/α_2 (for a fixed a_1 (Re_o)) will have the greatest amplitudes of longitudinal velocity a(Re). The dashed lines 1'-3' in Figure 4 illustrate the relationship of the amplitude of longitudinal velocity a at a point Re in the vicinity of branch II of the c.n.s. to the slope of k/α_2 for different a_1(Re_o). It is apparent that when $a_1 \gtrsim 0.05\%$ (corresponding to a_1 (Re_{II}) $\gtrsim$0.5%, perturbations with $k/\alpha_2 \approx$ 2 independent of a_1 will have the highest a . It would be of

interest to point out the increase in the selectivity of a boundary layer with an increase in a (the curves acquire increasingly sharper peaks).

Let us note that in a nongradient flow λ_x / λ_z does not depend on the Re point and is completely determined by the amplitude of the primary wave [16]. As calculations have shown, this is also valid for $\beta = 0.1$. In this case a change in a_1 (Re_o) from 0.02 to 0.04% is accompanied by a change in k/α_2 from 1.3 to 1.8, which remains constant with a further increase in a_1 (Re_o). The maximum rates of increase in σ (for specific initial a_1) are several times greater when $\beta = -0.1$ than when $\beta = 0.1$. Consequently, the amplitudes of the subharmonic and primary waves are equalized in a narrower range of Reynolds numbers when
$\beta < 0$.

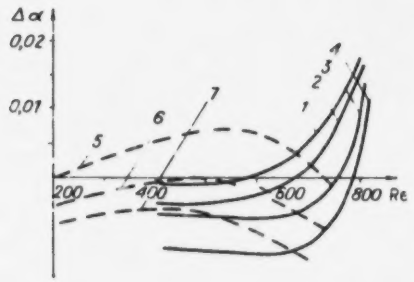


Figure 3. Desynchronization $\Delta\alpha$ for different values of b

0,18 (1), 0,17 (2), 0,151 (3), 0,122 (4), 0,22 (5), 0,2 (6), 0,175 (7). $Re_0 = 450$; $F_1(Re_0) = 1{,}15 \cdot 10^{-4}$.

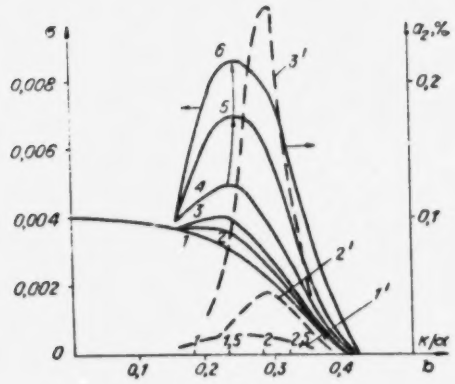


Figure 4. The b distribution of the local rate of increase of a subharmonic at point Re = 575 (solid lines)

1 — $a_1 = 0$, 2 — $a_1(Re_0) = 0{,}007\%$; 3 — 0,017%, 4 — 0,035%, 5 — 0,07%, 6 — 0,1%.

The b distribution of the amplitude of a subharmonic a at point Re = 705 (dashed lines)

$1' - a_1(\mathrm{Re}_0) = 0{,}035\%$, $2' -$
0,07%; $3 - 0{,}1\%$. $\beta = -0{,}1$, $\mathrm{Re}_0 = 450$, $F_1(\mathrm{Re}_0) =$
$= 1{,}15 \cdot 10^{-4}$, $a(\mathrm{Re}_0) = 0{,}001\%$.

Thus, as in the case of a flat plate, a broad k-spectrum of unstable three dimensional subharmonic perturbations occurs in gradient flows when the amplitude of a T-S plane wave is sufficiently low ($\sim$ 0.02%). The most "dangerous" are three dimensional modes which are linearly stable for all Re and whose rates of increase are an order of magnitude or more higher than maximum (for given β and F) linear rates of increase. The transversal wave number k of the most unstable mode is determined by the amplitude of the T-S plane wave and when $\beta < 0$ may depend on Re.

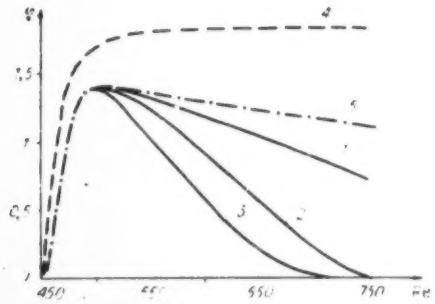
Let us examine the change in the energy of this subharmonic mode during the evolution process with respect to Re:

$$\frac{1}{2}\frac{1}{a^2}\frac{d}{dx}a^2 \approx \varepsilon \frac{|S| \cdot |h_+|}{|v_2|} a_1 \cos\varphi,$$

where

$$\varphi = \psi_1 - \psi_2 - \psi_3 + \arg S - \arg v_2 + \arg h_+.$$

The efficiency of energy transfer is determined by the nature of the relationship φ (Re a_1). Figure 5 gives the function φ for different primary wave amplitudes (curves 1-3). It also illustrates the curves arg h_+ (curve 4) and arg h_+ + arg S - arg v (curve 5). The presence of the imaginary parts S and v_2 is a synchronizing factor determined by viscosity properties. An increase in the amplitude of the primary wave
($a_1 > 0.02\%$) has a synchronizing effect by localizing the difference in phases $\psi_1 - \psi_2 - \psi_3$, thus ensuring maximum energy transfer to the given subharmonic component.



$1 - a_1(\mathrm{Re}_0) = 0{,}01\%$; $2 - 0{,}04\%$, $3 - 0{,}1\%$, $4 -$
arg h_+; $5 -$ arg h_+ + arg S − arg v_2. $\beta = -0{,}1$,
$\mathrm{Re}_0 = 450$, $F_1(\mathrm{Re}_0) = 1{,}15 \cdot 10^{-4}$ $b = 0{,}31$, $X = 500$.

Figure 5. The phase function φ for different primary wave amplitudes a (solid curves 1-3)

5. In the region of nonlinear interaction the most interesting effect of the pressure gradient (which is lacking when $\beta \geq 0$) is an increase in the rate of increase of σ of a fixed subharmonic mode with a decrease in its intensity a_1 (Re_0) when $\beta < 0$. This is valid with respect to the difference of the initial phase difference from zero. This relationship means that explosive amplification when $\beta < 0$ will occur in a narrow range of Reynolds numbers with a wide variation of initial low frequency perturbation parameters.

A direct comparison of the evolution of a triplet under conditions of different pressure gradients would not be trivial, because it would be impossible to determine the nondimensional values of the range of development (Re_0, Re_1) and the parameters k and f without establishing some correspondence between nondimensionalizing parameters for different β.

Figure 6 gives an example of calculating the evolution of a triplet on a physical range (x_0, x_1) for three values of β. x_0 and the velocities of external flow at this point U_e (x_0), which are assumed to coincide at it for all β, were chosen as common for the linear scale and the velocity scale. Then while when $\beta = 0$ points x_0 and x_1 correspond to Reynolds numbers of $Re_0^{(0)}$ and $Re_1^{(0)}$, when $\beta \neq 0$, they correspond to Reynolds numbers of $Re_0^{(m)} = Re_0^{(0)}$ and $Re_1^{(m)} = Re_1^{(0)} (Re_0^{(0)} / Re_1^{(0)})^{m}$. In the process the numerical values of F_1 and k at point x_0 will coincide for all β. As is apparent from Figure 6, a transition from faster flow ($\beta > 0$) to slower flow ($\beta < 0$) is accompanied by a significant rise in the rate of increase of the amplitudes of all components of the triplet.

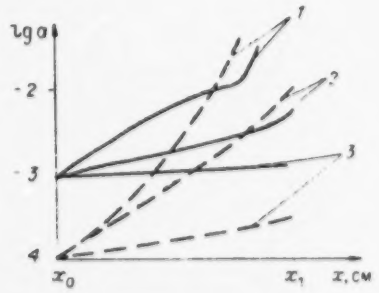


Figure 6. A comparison of the rates of increase of the components of the triplet a (solid lines) and a (dashed lines) for different values of β.

Thus, calculation results indicate that resonant interaction in triads in the presence of a pressure gradient is qualitatively similar to the case of nongradient flow. The amplitude of the plane wave is the primary factor determining mode discrimination and the evolution of the triplet. If the amplitude of the plane wave is greater than some sufficiently low threshold level, a broad spectrum of three dimensional subharmonic perturbations will be excited in all the flows considered. In slower flows the amplitudes of the primary wave and subharmonic

are equalized in a narrower range of Reynolds numbers and the rates of increase of both components are much higher. The above makes it possible to infer that a positive pressure gradient has a destabilizing effect on resonant interaction in triads, which makes the role of gradients decisive in subharmonic transitions on actual streamline bodies.

Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Soviet Academy of Sciences, Novosibirsk

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